**110.202 – Calculus III** 2/6/2017

Section 1.1

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# 1.1 Geometry of Euclidean Space

We'll start with a discussion of 2-D and 3-D euclidean space. In 2 dimensions we can represent space with a plane with coordinate axes. Points in the plane are represented by ordered pairs of real numbers $(x_1, y_1)$. We call $x_1$ the $x$-coordinate and $y_1$ the $y$-coordinate. In 3 dimensions, we'll also have a $z$-coordinate.

For notation, we say one-dimensional real space (the real line) is $\mathbb{R}$, two-dimensional real space (the set of ordered pairs of real numbers $(x, y)$) is $\mathbb{R}^2$, and three-dimensional real space (the set of all ordered triples $(x, y, z)$) is $\mathbb{R}^3$. We usually concentrate on these three, but we also have $\mathbb{R}^n$ for $n$-dimensional space.

Typically, we're first introduced to vectors as a direction and magnitude. While this intuition works in low dimensions, it turns out to be more convenient to think of vectors as a directed line segment beginning at the origin and ending at some point $(x, y, z)$ in space. We typically associate a vector with its ending point, and often write $\vec{a} = (a_1, a_2, a_3)$.

Standard basis vectors: Those of you with some linear algebra background will have seen this before, but it's basically a generalized way to talk about coordinate axes. In $\mathbb{R}^3$, we have three special vectors along the $x, y, z$ axes.

- $\vec{i} = (1, 0, 0)$ along the x-axis
- $\vec{j} = (0, 1, 0)$ along the y-axis
- $\vec{k} = (0, 0, 1)$ along the z-axis

(make note about orientation of axes)

We can represent any vector in $\mathbb{R}^3$ in terms of $\vec{i}, \vec{j}, \vec{k}$.

Equation of a line in the direction of a vector $\vec{v}$ and starting from the endpoint of the vector $\vec{a}$ is $\vec{\ell}(t) = \vec{a} + t\vec{v}$, where $t$ takes on all real values.

Equation of a line through the points $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$ is $(x, y, z) = (x_1 + (x_2 - x_1)t, y_1 + (y_2 - y_1)t, z_1 + (z_2 - z_1)t)$, where again, $t$ takes on all real values.