110.202 – Calculus III	February 13, 2017

Section #3

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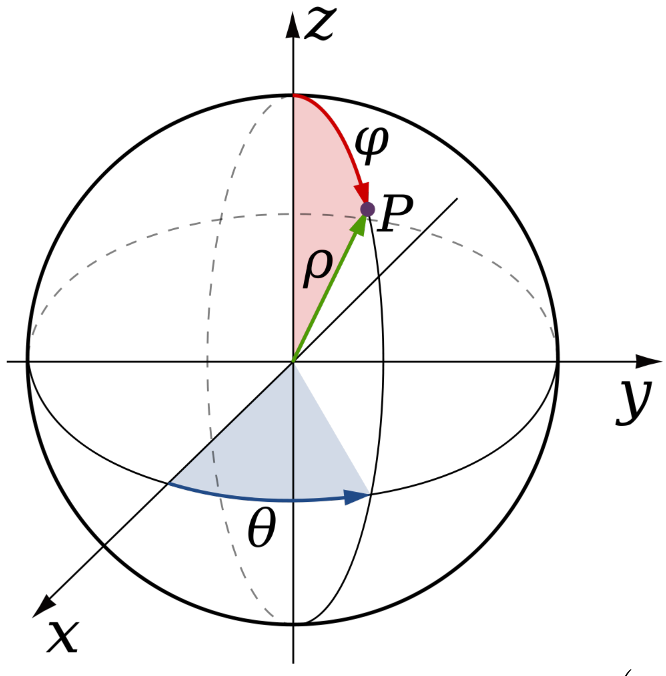
Covered: §1.4, §1.5, §2.1

1.4 – Cylindrical and Spherical Coordinates

Example problems: 1, 4, 10a, 14

Cylindrical Coordinates: $(x, y, z) \mapsto (r\cos\theta, r\sin\theta, z)$. Same as polar coordinates with an extra height component.

Spherical Coordinates: $(x, y, z) \mapsto (\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi)$. Here, ϕ is the colatitude (complementary angle to the latitude), θ is the longitudinal angle, and ρ is the radius. As a result, $\phi \in [0, \pi], \theta \in [0, 2\pi], \rho \geq 0$.



Using the dot product, we can express $\phi = \cos^{-1}\left(\frac{1}{\|v\|}\vec{v} \cdot \hat{k}\right)$. This is because $\vec{v} \cdot \hat{k} = \|\vec{v}\|\|\hat{k}\| \cos\phi$ since we define ϕ to be colatitude.

1.5 – n-Dimensional Euclidean Space

Example problems: 2, 4, 9

Vectors

Inner products are the same as dot products in $\mathbb{R}^3$, same four properties as seen before.

Cauchy-Schwarz still holds. Last time, in $\mathbb{R}^3$, we used the law of cosines, but there is also an algebraic proof.

Proof. Let $a = \vec{y}\cdot\vec{y}, b = -\vec{x}\cdot\vec{y}$. If $a = 0$, then $\vec{y} = 0$, and the inequality holds as usual. So assume $a \neq 0$. Then, $0 \geq (a\vec{x}+b\vec{x})\cdot(a\vec{x}+b\vec{x}) = a^2\vec{x}\cdot\vec{x}+2ab\vec{x}\cdot\vec{y}+b^2\vec{y}\cdot\vec{y}$. Then, $(\vec{y}\cdot\vec{y})^2\vec{x}\cdot\vec{x}-(y\cdot y)(x\cdot y)^2 \geq 0$. Dividing by $y \cdot y$ gives $(x \cdot y)^2 \leq (x \cdot x)(y \cdot y)$ and we just take square roots and we're done.

Similarly, triangle inequality still holds in $\mathbb{R}^n$.

Matrices

Last time we focused on 2×2 and 3×3 matrices. You can also have general $m\times n$ matrices. Recall that m is the number of rows and n is the number of columns in the matrix. You can multiply matrices A, B if A is $a \times b$ and B is $c \times d$ as long as $b = c$. The resulting matrix AB is an $a \times d$ matrix. Incidentally, this is the first example most people see of noncommutative operations, i.e. those where $AB \neq BA$.

In general, an $m \times n$ matrix M is a function from R^n to R^m. You can see that if you multiply M by a $n \times 1$ vector in R^n you get a $m \times 1$ vector in $\mathbb{R}^m$. In fact, matrix multiplication is a linear transformation. This means $M(a\vec{x} + \vec{y}) = aM\vec{x} + M\vec{y}$ for any constant a.

If an $n\times n$ matrix A is invertible, then a matrix A^{-1} exists such that $AA^{-1} = I_n$, where I is the identity matrix. A matrix is invertible if and only if its determinant is not zero, as $\det(A) = \frac{1}{\det A^{-1}}$.

2.1 – Geometry of Real-Valued Functions

Example problems: 1, 5

Since this hasn't been covered in lecture yet, I'll give a brief overview of the following topics: Graph of a function, Level Sets/Curves/Surfaces, Sections of graphs. I'll be using book examples for them.