110.202 – Calculus III March 7, 2017

Section #5

TA: David Li

Covered: Exam Questions, §3.1, §3.2, §3.3

3.1 – Iterated Partial Derivatives

HW: 1, 4, 7, 12, 22, 32

- Iterated Partials
- Equality of Mixed Partials – Always for C^2 functions
- Example 4 from text

3.2 – Taylor's Theorem

HW: 1, 2, 6, 10

- Recall single-variable case – second order Taylor expansion of $f(t)$ centered at t_0 is

$$T(t) = f(t_0) + f'(t_0)(t - t_0) + \frac{f''(t_0)}{2}(t - t_0)^2.$$

- Similar idea in multiple variables – first order Taylor expansion of $f(x)$ centered at x_0 is

$$T(x_0 + h) = f(x) + \sum_{i=1}^{n} h_i \frac{\partial f}{\partial x_i}(x_0)$$

- Second order expansion of $f(x)$ centered at x_0 is

$$T(x_0 + h) = f(x) + \sum_{i=1}^{n} h_i \frac{\partial f}{\partial x_i}(x_0) + \frac{1}{2}\sum_{i,j=1}^{n} h_i h_j \frac{\partial^2 f}{\partial x_i \partial j}(x_0)$$

- Recall tangent plane which is a first order approx

$$f(x, y) \approx f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

. Second order approximation

$$\begin{aligned} f(x, y) \approx\ & f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) \\ & + \frac{1}{2}(f_{xx}(x_0, y_0)(x - x_0)^2 + 2f_{xy}(x_0, y_0)(x - x_0)(y - y_0) + f_{yy}(x_0, y_0)(y - y_0)^2) \end{aligned}$$

- Example 3 from text

3.3 – Extrema of Multivariable Functions

HW: 6, 8, 18, 26, 28, 30, 31, 38, 44

- Critical points occur when gradient is 0.
- Hessian Matrix of a function $f\colon \mathbb{R}^n \to \mathbb{R}$ is the $n \times n$ square matrix H where $H_{i,j} = \frac{\partial^2 f}{\partial x_i \partial x_j}$
- Second derivative test for functions of two variables.

$$\begin{aligned} f_{xx} < 0 \text{ and } f_{xx}f_{yy} - f_{xy}^2 > 0 &\implies \text{local maximum at } (a,b). \\ f_{xx} > 0 \text{ and } f_{xx}f_{yy} - f_{xy}^2 > 0 &\implies \text{local minimum at } (a,b). \\ f_{xx}f_{yy} - f_{xy}^2 < 0 &\implies \text{saddle point at } (a,b). \\ f_{xx}f_{yy} - f_{xy}^2 = 0 &\implies \text{inconclusive test at } (a,b). \end{aligned}$$

- Global maxima/minima (analagous to the extreme value theorem)
- Steps to find global min/max
  (a) Find all critical points inside the region
  (b) Find all critical points on the boundary (if the boundary is not smooth make sure to consider "corners")
  (c) Find the value at all the critical points
  (d) Find the largest and smallest